

Exercise 17

For the following exercises, use each pair of functions to find $f(g(x))$ and $g(f(x))$. Simplify your answers.

$$f(x) = \frac{1}{x-4}, \quad g(x) = \frac{2}{x} + 4$$

Solution

Compute $f(g(x))$ by plugging the formula for $g(x)$ where x is in the formula for $f(x)$.

$$\begin{aligned} f(g(x)) &= \frac{1}{\left(\frac{2}{x} + 4\right) - 4} \\ &= \frac{1}{\frac{2}{x}} \\ &= 1 \times \frac{x}{2} \\ &= \frac{x}{2} \end{aligned}$$

Compute $g(f(x))$ by plugging the formula for $f(x)$ where x is in the formula for $g(x)$.

$$\begin{aligned} g(f(x)) &= \frac{2}{\left(\frac{1}{x-4}\right)} + 4 \\ &= 2 \times \frac{x-4}{1} + 4 \\ &= 2(x-4) + 4 \\ &= 2x - 8 + 4 \\ &= 2x - 4 \end{aligned}$$